### D.Phil. /Ph.D. Course Work in Mathematics (in partial fulfilment for the award of the degree of D.Phil./Ph.D.)

### One Semester Course

There shall be 2 Core Papers:

MAT801 Research Methodology

MAT802 Communication skill and Seminars

There shall be 2 Elective Papers that a candidate shall have to take from the area in which the research shall be carried out.

Each paper shall be of 100 marks and 2 credits.

Total 8 credits shall be assigned to the D.Phil./Ph.D. Course Work.

### Course Structure

Cours Code/Name of the Course/Credits (L-T-P-C)

MAT801	Research Methodology	1-0-2-2
MAT802	Communication skill and Seminars	0-1-2-2
MAT851	Algebra	1-1-0-2
MAT852	Analysis	1-1-0-2
MAT853	Topology	1-1-0-2
MAT854	Group Theory	1-1-0-2
MAT855	Structures on Manifolds	1-1-0-2
MAT856	Dynamics of Fluids in Porous Media	
	& MHD Flows	1-1-0-2
MAT857	Lattice Theory	1-1-0-2
MAT858	Mathematical Modelling	
	and its Applications	1-1-0-2
MAT859	Commutative Algebra	1-1-0-2
MAT860	Topics in Number Theory	1-1-0-2
MAT861	Theory of Rings and Modules	1-1-0-2
MAT862	Quantum Information Processing (QIP)	1-1-0-2
MAT863	Wavelet Analysis	1-1-0-2

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## MAT 801: Research Methodology

Learning of LaTeX/TeX, WORD, PDF including formatting and preparation of manuscripts for journals, peer-review process, survey of literature, MathSciNet Database, Publication Ethics, Plagiarism (URKUND)

## MAT 802: Communication skills & Seminars

Skill development to communicate with a group of people, slide-shows/power point presentations, Elementary idea of various search engines

### MAT851 Algebra

Review of basic concepts in algebra and linear algebra, Fundamental Theorem of homomorphism, Isomorphism Theorems, Cayley's Theorem, Sylow Theorems and its applications, Maximal and prime ideals, Principal ideal domains, Unique factorization domains and Euclidean domains, Finite fields

### References

- 1. M. Suzuki, Group Theory, Vol 1 Springer, New York, 1980
- 2 . M. Artin, Algebra, PHI, New York, 1988

3. D.J.S. Robinson, A course in the theory of groups, Graduate text in Mathematics, No. 80, Springer, New York 1996.

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## MAT854 Group Theory

Groups defined by generators and relations, Symmetric and alternating groups and their automorphism groups, Free product with amalgamations, finite p-groups, Central product of groups, Extra special p-groups, sharply 2-transitive groups

### References

- 1. M. Suzuki, Group Theory, Vol 1 Springer, New York, 1980
- 2 . M. Suzuki, Group Theory, Vol 2 Springer, New York, 1986
- 3. D.J.S. Robinson, A course in the theory of groups, Graduate text in Mathematics, No. 80, Springer, New York 1996.

## **MAT855 Structures on Manifolds**

Complex Manifolds: Almost complex manifolds, Nijenhuis tensor, eigen values of an almost complex structure, existence theorem and integrability condition of an almost complex structure, complex manifolds, almost Hermitian manifolds, some well known classes of almost Hermitian manifolds and their curvature properties, Kaehler and para-Kaehler manifolds.

Contact Manifolds: Contact metric manifolds, almost contact manifolds, torsion tensor of an almost contact metric manifold, killing vector fields, properties of structure tensor  $\phi$ , curvature properties of contact metric manifolds, para-contact manifolds. K-Contact Manifolds: Characterizations of K-contact manifolds, curvature properties, sectional curvature, locally symmetric and semi-symmetric K-contact manifolds. Sasakian manifolds: Curvature properties,  $\phi$ -sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor on a Sasakian manifold.

### Refernces:

1. K. Yano and M. Kon; Structures on Manifolds, World Scientific, 1984.

2. D. E. Blair; Riemannian Geometry of Contact and Symplectic Manifolds (Progress in Mathematics, Vol. 203), Birkhauser, Boston, 2002.

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## MAT852 Analysis

Review of basic concepts in analysis (real and complex):, Significance and applications of Mean value theorem, Inverse function theorem and implicit function theorem, Lebesgue measure and outer measure, Lebesgue integration, Cauchy-Riemann equations, Cauchy's Theorem, Morera's Theorem, Laurent series expansion, maximum modulus Theorem, Conformal mappings

### References

- 1.T.M: Apostol; Mathematical Analysis, Narosa Publishing House, New Delhi, 2002.
- 2. H. L. Royden and P. M. Fitzpatrick, Real Analysis, (Fourth edition), Prentice Hall of India, 2010.
- 3.1. J. B. Conway, Functions of One Complex Variable, Narosa Publishing House, New Delhi, 2002.
- 4. S. Ponnusamy and H. Silverman, Complex Variables, Birkhäuser, Inc., Boston, MA, 2006

### **MAT853 Topology**

Review of point-set topology (basic notions including subspace, product quotients), Statements of Urysohn's lemma, Tietze's extension theorem, Embedding lemma, Urysohn's metrization theorem, Preliminaries in topological groups, classical matrix groups.

### References

- 1. J.L. Kelley, General Topology, Von Nostrand 1995
- 2. G.F. Simmons, Introduction to Topology and modern analysis

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## MAT857 Lattice Theory

Partially ordered sets (posets), blocks, Duality principle, lower cone, upper cone, and their properties, Hasse diagram of a poset, Chain and chain conditions, refinement of a chain, equivalent chain, composition series, length of composition chain.

Definition and examples of lattices, properties of lattice, sublattice, lattice morphism, isomorphim, isotone mapping, closure operator, ideal and filter, atom, atomic lattice, complete lattice, fixed point theorem, Kogalovskii's theorem, complement and its properties, complemented lattice, Dilworth theorem, modular lattice, characterization of modular lattice, Valuation of lattice, positive valuation, metric lattice, distributive lattice, characterisation of distributive lattice, ortholattice, orthomodular law, orthomodular lattice.

### References

(1) L. A. Skornjakov, Elements of lattice theory, Hindustan Publishing Corporation(India),

(2) D.E. Rutherford, Introduction to lattice theory, Oliver and Boyd Ltd. ,Tweeddale Court, Edinburg, 1965.

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## MAT856 Dynamics of fluids in Porous Media & MHD Flows

Review of basic concepts in fluid mechanics:- Principles of conservations of mass, momentum (linear & angular) and their significance, Concept of Stresses and rate of strains, Newtonian and non-Newtonian fluids, Constitutive equations and their importance, Navier-Stokes equations of motion, Boundary conditions in fluid mechanics and their importance, Concept of dynamical similarity and dimensionless numbers and their significance in the fluid dynamics, Stokes flow and the introductory concept of the boundary layer theory, Cell methods, Introduction to theory of micropolar fluids.

Mechanics of fluid flow through porous medium: Porosity and permeability, seepage effective viscosity, Darcy's law, Brinkman's equation, Darcy's Forchhiemer's equation, Darcy- Lapwood- Forchhiemer's- Brinkman's equation, Boussinesq approximation, Convection, filtration.

Introduction of Magnetohydrodynamics (MHD), MHD approximations.

#### Reference books:

1. Z.U.A. Warsi; Fluid Dynamics, CRC Press (2005)

2. J. Happel and H. Brenner; Low Reynolds Number Hydrodynamics, Academic Publishers group, (1983)

3. D.A. Nield and A. Bejan; Convection in Porous Media, Springer, New York (2006).

4. G. Lukaszewich; Micropolar fluids: Theory and Applications, Birkhauser (1999).

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### MAT858 Mathematical Modelling and its Applications

Mathematical Models: Single Species Growth model, Malthusian Growth Model, Logistic Growth Model: Insect out Break Model.

Biological Interactions; Models for Interacting Species: Predator-Prey interaction: Lotka-Volterra Models, Dynamics of the Simple Lotka-Volterra Models, Role of Density Dependence in the Prey, Analysis of a Predator-Prey Model with Limit Cycle Periodic Behaviour: Parameter Domains of Stability; Functional Response; Some Predator- Prey models; Predation in Natural Systems, Two Patch Predator-Prey Model.

Competition: Lotka-Volterra Models, Extension to Lotka-Volterra Models, Competition for fixed resources and renewable resources, Diffusion Models.

Delay Models, Stage-Structured Models.

### **Books Recommended:**

- 1. J. D. Murray, 2002, Mathematical Biology: An Introduction, Springer
- 2. Alan Hastings, 1997, Population Biology: Concepts and Models, Springer-Verlag
- 3. P. Turchin, 2003, Complex Population Dynamics: A Theoretical/Empirical Synthesis, Princeton University Press.

West Charles Cology, M., 4. H. I Freedman, 1980, Deterministic Mathematical Models in Population Ecology, M., dekker, New York.

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## MAT859 COMMUTATIVE ALGEBRA

Tensor product of Modules, Flat modules, homology of complexes, derived functors.Localisation, local rings, nil and Jacobson radicals, Nakayama's lemma, prime spectrum of rings. Noetherian modules, primary decompositions, affine algebraic sets, integral xtensions,

Hilbert's nustellensatz. Artinian modules, Discrete valuation rings, Dedekind domains.

### References:

- 1. N. Jacobson, Basic Algebra, vol. 2, Hindustan Publishing Corporation, 1994.
- 2. M.F. Atiyah and I. G. MacDonald, Introduction to Commutative Algebra , Addison Wesely Publishing Company Inc. 1969.
- 3. N.S. Gopalkrishnan, Commutative Algebra, Oxonian Press, 1984.

### MAT860 Topics in Number Theory

Primes of special form, Jacobi and Legendre symbols, Gauss and Jacobi sums, Cubic and biquadratic reciprocity.

Equation over finite fields, Diiophantine equations.

Absolute value on fields, p-adic number fields, quadratic forms over Q and Q<sub>p</sub> Hasse-Minkowaski theorem.

The Riemann zeta function, Dirichlet series and characters, primes in arithmetic progression.

### References

- 1. K. Ireland and M. Rosen, A Classical Introduction to Modern Number Theory, Springer -Verlag, 1994.
- 2. J. P. Serre, A Course in Arithmetic, GTM-7, Springer-Verlag 1973.

3. M. Hindry, Arithmetics, Springer-Verlag, 2011.

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#### THEORY OF RINGS AND MODULES **MAT861**

Review of basic concepts in rings and modules, nilpotent and nil ideals, radicals, chain conditions on rings and modules, prime rings, primitive rings.

Annihilator, socle, essential and small submodules, properties of singular and nonsingular modules, injective modules and injective envelopes, projective modules and projective covers.

### References:

- (1) F. W. Anderson and K. R. Fuller, Rings and Category of Modules, Graduate texts in mathematics, Vol 13, Springer-Verlag Inc., New York, 1974.
- (2) T. Y. Lam, A First Course in Noncommutative Rings, Graduate texts in mathematics: 131, Springer-Verlag New York, Inc., 1991.
- (3) K. R. Goodearl, Ring Theory: non singular rings and modules, Marcel Dekker, Inc. New York and Basel, 1976.
- (4) Paul E. Bland, Rings and their Modules, Walter de Gruyter GmbH & Co. KG, Berlin/New York, 2011.
- (5) O. Zariski and P. Samuel, Commutative Algebra, Volume I, D. Van Nostrand Company, Inc., 1958.
- (6) T. Y. Lam, Lectures on Modules and Rings, Graduate texts in mathematics: 189, Springer-Verlag, New York, Inc., 1999.

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#### MAT862 Quantum Information Processing (QIP)

Shannon Entropy, data compression, coding theory, noisy channels and mutual Information.

Postulates of quantum theory, composite Systems. Entanglement measures, entanglement of formation and distillable entanglement.

Bell states, EPR paradox, Bell's inequalities, GHZ paradox and Hardy's nonlocality. Quantum contextuality and quantum steering.

Quantum teleportation, quantum dense coding, measures of quantum correlations, quantum cloning and distinguishability, LOCC. Quantum protocols, quantum cryptography, device independent security proof.

Quantum algorithms, Shor's factorization algorithm. Graph states, Quantum error correction, CSS codes, stabilizer code formalism, Clifford codes. Fault-tolerant quantum computation.

## Books Recommended:

1. M.A. Nielsen and I. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, Cambridge, 2000.

2.

### Further Reading:

- 1. Phillip Kaye, Michele Mosca & Raymond Laflamme, An Introduction to Quantum Computing, Oxford University Press 2007.
- 2. Edited by Daniel A. Lidar and Todd A. Brun, Quantum Error Correction, 20013.
- 3. G. Benenti, G. Casati and Giuliano Strini, Principles of Quantum Computation and Information, Vol. 1: Basic Concepts, Vol II: Basic Tools and Special Topics, World
- 4. Asher Peres, Quantum Theory: Concepts and Methods, Kluwer Academic Publ., Dordrecht,
- 5. J. Preskill, Lecture notes on Quantum Computation, Physic 219/Comp. Sc.219 Course, available in http://theory.caltech.edu/people/preskill/ph229.
- 6. J. J. Sakurai, Modern Quantum Mechanics, 2nd eds., Addison-Wesley, ISE Reprint, 1999.
- 7. H. K. Lo, S. Popescu and T. Spiller, Introduction to Quantum Computation and Information,
- 8. Ryszard Horodecki, Paweł Horodecki, Michał Horodecki, and Karol Horodecki, Quantum entanglement, Rev. Mod. Phys. 81, 865 (2009).

9. Nicolas Brunner, Daniel Cavalcanti, Stefano Pironio, Valerio Scarani, and Stephanie Wehner, Tay Cathon Cathon PSZ Bell nonlocality, Rev. Mod. Phys. 86, 419 (2014).

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# MAT863 Wavelet Analysis

Orthonormal bases for  $L^2(\mathbb{R})$ , Smooth projections on  $L^2(\mathbb{R})$ , Local sine and cosine bases and construction of some wavelets, Lemarie -Meyer wavelets.

The construction and better estimates for the smoothness of compactly supported wavelets. Spline wavelets on the real line, Orthonormal bases of piece wise linear continuous functions for  $L^2(\mathbb{T})$ , Periodization of wavelets.

Characterizations of low pass filters and scaling functions, Non existence of smooth wavelets in  $H^2(\mathbb{R})$ , Generalized Multiresolution analysis, Wavelet packets, Frames, The Balion-Low theorem for frames, Frames from dilation and translations. Smooth frames for  $H^2(\mathbb{R})$ .

Wavelet transform, Hilbert transform, Wavelets and Multiresolution Analysis in higher

## **Books Recommended:**

- I. E. Hernandez and G. Weiss, A First Course on Wavelets, CRC press.
- II. David F. Walnut, An Introduction to Wavelet, Birkhauser publication.
- P. Wojtaszczyk, A Mathematical Introduction to Wavelets, London Mathematical III. Society. Cambridge University Press.
- Ole Christensen, An Introduction to Frames and Riesz Bases, Birkhauser publication. IV.

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